AutoDots Maths Paper 1

Pure mathematics and modelling

Time allowed: 45 minutes **Total: 30 marks**

This original AS-level practice paper has five questions. Answer them all. Show your working. A calculator is allowed. Give exact answers unless a question asks for an estimate.

Question 1 Indices and algebra

(a) For $x > 0$, simplify $\dfrac{12x^{-2}y^{3}}{3x^{1/2}}$. Give your answer using positive indices. [3 marks]

(b) Solve $2^{2t} - 5(2^{t}) + 4 = 0$ for real t. [3 marks]

Question 2 Two intersecting curves

The curve C has equation $y = x^2 - 2x - 3$. The line L has equation $y = x - 3$. **(a)** Find the coordinates of both points where C and L meet. [3 marks]

(b) Find the equation of the tangent to C at the intersection with the larger x-coordinate. [3 marks]

Turn over for Question 3.

Question 3 A transformed cooling curve

A technician measures the temperature of a sealed sample. Figure 1 shows the model $T = f(t)$ during the first six minutes. For this question, treat the marked readings as exact.

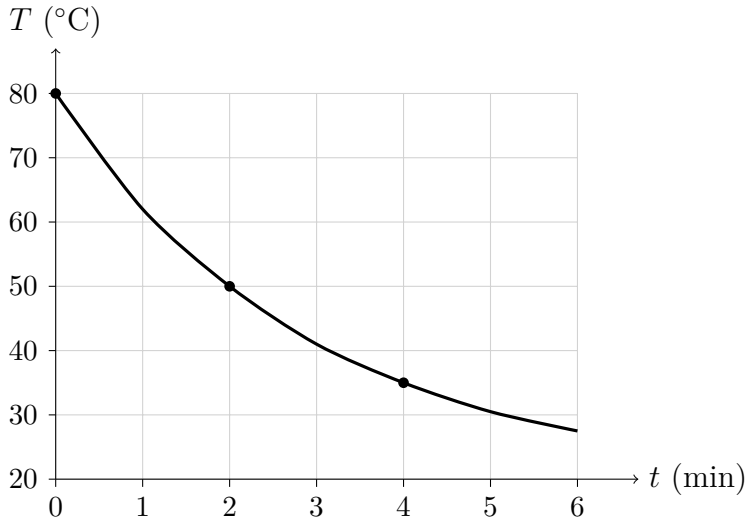


Figure 1. Cooling model. The dots mark readings at 0, 2 and 4 minutes. **(a)** Use the marked readings to calculate the average rate of temperature change over the first four minutes. Give units. [3 marks]

(b) A second sensor reports $S(t) = f(t - 2) + 4$ for $2 \le t \le 8$. Using Figure 1, find $S(4)$ and explain both transformations. [3 marks]

Question 4 A box with an open top

Squares of side x cm are cut from the corners of a sheet measuring 20 cm by 12 cm. The sides fold up to form a box. **(a)** Write its volume V as a function of x and state the physical domain. [2 marks]

(b) Find the value of x that maximises V. Give an exact answer and justify that it gives a maximum. [5 marks]

Question 5 A model and its residuals

A machine fills packets. The table gives the mass m dispensed after t seconds.

Time t (seconds)	2	4	6	8
Mass m (grams)	19	32	45	53

(a) Fit a straight line through the first and third readings. Write down its equation. [3 marks]

(b) Use your line to predict the mass at 8 seconds. Find the residual (observed minus predicted). [2 marks]

End of paper